O K L A H O M A S T A T E U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

ECEN/MAE 5713 Linear Systems
Spring 2011
Midterm Exam #2

DO ALL FIVE PROBLEMS

Name : ______________________________

E-Mail Address:_______________________________________

Problem 1:

a) Show that the set of all real $n \times n$ matrices with the usual operation of matrix addition and the usual operation of multiplication of matrices by scalars constitutes a vector space over the reals, $\Re$, denoted by $\left(\Re^{n \times n}, \Re\right)$. Determine the dimension and a basis for this space.

b) Is the above statement still true if $\Re^{n \times n}$ is replaced by the set of nonsingular matrices?

Justify your answers.

Problem 2:
Let

$$S = \left\{ x \in \Re^3 \mid x = \alpha \begin{bmatrix} 0.6 \\ 1.2 \\ 0.0 \end{bmatrix} + \beta \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}, \alpha, \beta \in \Re \right\},$$

find the orthogonal complement space of S, $S^{\perp}(\subset \Re^3)$, and determine an orthonormal basis and dimension for $S^{\perp}$. For $x = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T (\in \Re^3)$, find its direct sum representation (i.e., x_1 and x_2) of $x = x_1 \oplus x_2$, such that $x_1 \in S, x_2 \in S^{\perp}$.

<u>Problem 3</u>:

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix},$$

what are the rank and nullity of the above linear operator, A ? And find the bases of the range spaces and the null spaces of the operator, A ?

Problem 4:

Extend the set

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ 4 \end{bmatrix}$$

with constraints $3x_1 = 3$ to form a basis in $(\Re^4, \Re)$

Problem 5:

Show if the following two sets

$$\begin{bmatrix} 2 & -2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -4 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$$

span the same subspace V of $\left(\Re^{2\times 2}, \Re\right)$.